# Synchrophasor Measurement based Assessment of Voltage Stability in Power System 

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#### Abstract

In this paper, voltage stability assessment methodology using Phasor Measurement Unit (PMU) has been proposed. For observability of the power network, PMU has been installed at the buses of the system. Due to higher installation cost of PMU, Integer Linear Programming (ILP) has been used to optimize the number of PMUs at the buses. PMU's data are used for voltage stability assessment with the help of L-Index. As the PMU gives real time voltage and current phasors and L-index is dependent on voltage and admittance values, thus the L-index so obtained can be used as real time voltage stability indicator. All the results have been tested on IEEE 30-bus test system.


## 1. INTRODUCTION

Power System supports the world economy to a great extent. Therefore, stability of power system is a key concern for power engineers. For this purpose, it is important to analyze the system for every instant of time, so use of real-time measurements are necessary. Voltage stability is one of the most important areas for engineers to maintain the operation of power system within contractual and steady voltage limits before and after any disturbances. These disturbances are, may be, sudden loss of generation or lines, or changing loads, which affects the operating point of system and frequency. So, it is necessary to rapidly monitor and adjust to system changes to attain a new operating point or an equilibrium point keeping generation to load balance. This ability of system is the goal of voltage stability assessment and control [1]. Series of blackouts encountered in recent years in power system, have been occurred because either of voltage or angle instability or both together was not detected within time and progressive voltage or angle instability further degraded the system condition, because of increase in loading [2, 3]. So, synchronized phasor measurements are very important for wide area measurement systems used in advanced power system monitoring, protection, and control applications.

Several algorithms and approaches have been published in the literature for the optimal placement of PMUs in power system.

The authors in $[4,5]$ developed an optimal placement algorithm for PMUs by using integer linear programming. Communication Infrastructure and Islanding based optimal placement of PMUs have been documented in [6-8] and [9, 10] respectively. Voltage stability ranking for OPP has been proposed in [11]. Authors in [12] presented a multi-objective function to optimize the number of PMUs and maximize the observability by using Binary Gravitational Search Algorithm (BGSA). In this paper, a voltage stability index has been proposed as an application of PMUs. A static voltage stability index has been proposed in [13] which has been used the synchrophasor technology to early detection of impending voltage instability.
In this paper, PMUs are placed at strategically obtained location such that minimum number of PMU's can make all load buses observable and provide the on-line voltage phasors to predict the voltage stability. For this purpose, a formulation of optimal PMU placement problem is done by Integer Linear Programming. These PMUs provide voltage phasors of all load buses at very small intervals. Thus the rate of change of voltage angle due to change in load with respect to successive intervals of time and this can be used as the VSP to estimate the voltage stability. As the PMU gives real time voltage and current phasors, thus voltage stability predictor can be used for on-line voltage stability prediction. The voltage stability assessment problem using PMU is more efficient and can be used in practice.

## 2. MATHEMATICAL FORMULATION OF PROPOSED METHOD

### 2.1. Optimal PMU placement methodology

A novel objective function has been proposed to optimize the number of PMUs which provides the full observability of the power network. OPP results consider the presence of zero injection buses into the system [7]. The proposed objective function can be written as follows:

Minimize $\quad \sum_{i=1}^{n} w_{i} z_{i}=\sum_{i=1}^{n} \frac{z_{i}}{\left(c_{c l} * N C_{i}\right)+c_{p}}$
Subject to: $\quad f=A . Z \geq 1$
Connectivity matrix ( $A$ ) defines the interconnection of system buses by transmission lines. The entries in $A$ are defined as follows:
$A_{i j}= \begin{cases}1 & \text { if } i=j \\ 1 & \text { if } i \text { and } j \text { are connected } \\ 0 & \text { otherwise }\end{cases}$
where $z_{i}$ is the elements of vector $Z$, which represents the status of the installation of a PMU at bus i. if $z_{i}=1$, it means PMU is installed at bus i , otherwise $z_{i}=0 . \mathrm{p}$ is the total number of PMUs. $c_{c l}$ and $c_{p}$ represent the cost of channel and cost of PMU respectively. $N C_{i}$ is the numbers of channel at bus $i$. Equation (1) can also use as a multiobjective function which minimize the PMU and maximize the observability of the system. But in this paper, Equation (1) used only to optimize the PMUs. In equation (1), $w_{i}$ is the weight factor and element of column vector W , which represents the inverse of the cost of PMU with respect to number of channels (branches) connected to bus i. $w_{i}$ is defined as follows:

$$
\begin{equation*}
w_{i}=\frac{1}{\left(c_{c l} * N C_{i}\right)+c_{p}} \tag{4}
\end{equation*}
$$

### 2.2 L-index as a Voltage Stability Assessment Tool

Consider a single generator and single load system, as shown below in Fig. 1, for assessment of voltage.


Fig. 1: Single generator and single load system

Since, current $I_{D}$ of load bus for two bus system shown above, is given by,
$I_{D}=\frac{S_{D}^{*}}{V_{D}^{*}}=V_{D} Y_{Q}+\left(V_{D}-V_{G}\right) Y_{L}$
$S_{D}^{*}=V_{D}^{2} Y_{Q}+V_{D}^{2} Y_{L}-V_{D}^{*} V_{G} Y_{L}=V_{D}^{2} Y_{11}+V_{O} V_{D}^{*} Y_{11}$
Here, $Y_{11}=Y_{Q}+Y_{L}$ and $V_{O}=-\left(\frac{Y_{L}}{Y_{Q}+Y_{L}}\right) V_{G}$
Now L-index is given in equation 7 :
$L=\left|\frac{S_{D}^{*}}{V_{D}^{2} Y_{11}}\right|=\left|1+\frac{V_{o}^{0}}{V_{D}}\right|$
When the load is zero, i.e. $\mathrm{SD}=0$, then $L=0$, if the voltage at bus 1 collapses, $\mathrm{L}=1$. The detailed information of L -index has been given in [14].

## 3. RESULTS AND DISCUSSION

In this paper, OPP problem has been solved using Integer Linear Programming (ILP) under MATLAB to determine the minimum number of PMUs such that entire system is observable. The solution of the problem yields the minimum number of PMUs which will make the system fully observable. Results for IEEE 14-bus and IEEE 30-bus test systems are shown in Table 1. It can be seen from this table that 4 locations of PMUs for IEEE-14 bus and 10 locations for IEEE 30 -bus test systems are sufficient to produce fully observable measurements.

Table 1: OPP Results for IEEE test systems

| System | Number of <br> PMUs | Location of PMUs (bus <br> number) |
| :---: | :---: | :---: |
| IEEE 14-bus | 3 | $2,8,10,13$ |
| IEEE 30-bus | 7 | $3,5,10,12,18,23,27$ |

In this paper, L-index method has been used to verify the voltage stability of the different load buses of the system. The effect of increase in load at all load buses of system, from base case, on voltage stability is assessed by using L-index. For IEEE 30 -bus test system, the voltage magnitude, angle and Lindex for individual buses and Lmax for system are given in Table 2 shown below. From Table 2, it is clear that when total loading of system is increased to the loading nearer to the collapse point, L-index of some buses is still closer to the zero (0) and for some buses L-index is approaches towards unity (1). The variation in System Stability Indicator (Lmax) with increase in loading at individual bus of IEEE-30 bus system is shown in Fig. 2 to 4 for increase in loading at load bus-21, bus-26 and bus- 30 of IEEE 30 -bus test system respectively.


Fig. 2: System Stability Indicator (Lmax) to Loading at Bus 21

Table 2: Voltage phasor and L-index and $L_{\text {max }}$ at voltage collapse point due to increase in total load

| bus <br> no. | Voltage <br> Magnitude | Voltage Angle | Value of L-index |
| :---: | :---: | :---: | :---: |
| 3 | 0.8711 | -28.0027 | 0.0636 |
| 4 | 0.8570 | -35.5800 | 0.0759 |
| 6 | 0.8837 | -42.9663 | 0.0669 |
| 7 | 0.8868 | -49.3943 | 0.0834 |
| 9 | 0.8821 | -55.2950 | 0.178 |
| 10 | 0.8089 | -62.5413 | 0.3605 |
| 12 | 0.8781 | -59.8488 | 0.2146 |
| 14 | 0.8174 | -63.6845 | 0.332 |
| 15 | 0.7936 | -63.9829 | 0.3689 |
| 16 | 0.8231 | -62.1010 | 0.3152 |
| 17 | 0.7940 | -63.3538 | 0.3815 |
| 18 | 0.7517 | -66.8613 | 0.4731 |
| 19 | 0.7391 | -67.6807 | 0.5086 |
| 20 | 0.7532 | -66.6159 | 0.4772 |
| 21 | 0.7580 | -64.7593 | 0.4533 |
| 22 | 0.7586 | -64.7135 | 0.4519 |
| 23 | 0.7382 | -65.9336 | 0.4807 |
| 24 | 0.6970 | -66.9351 | 0.5759 |
| 25 | 0.6689 | -66.6173 | 0.6485 |
| 26 | 0.5855 | -69.6006 | 0.8703 |
| 27 | 0.6931 | -64.7980 | 0.5974 |
| 28 | 0.8714 | -45.2970 | 0.0954 |
| 29 | 0.5718 | -73.5453 | 0.9303 |
| 30 | 0.5008 | -81.9654 | 0.9848 |
| Lmax (system stability indicator) |  |  |  |



Fig. 3: System Stability Indicator (Lmax) to Loading at Bus 26


Fig. 4: System Stability Indicator (Lmax) to Loading at Bus 30
(Lig. Sys) to Loading at Bus 26

In Table 3, the voltage magnitude, voltage angle and L-index of IEEE 30-bus at all load buses are given when load at bus 21 is increased and approaches nearer to collapse point. Similarly Table 4 and Table 5 shows the Voltage magnitude, voltage angle and L-index at all the IEEE 30 -bus test system load buses when load at bus 26 and bus 30 of IEEE 30 -bus test system is increased and approaches nearer to collapse point respectively.

Table 3: Voltage phasor, L-index and Lmax at voltage collapse point due to load increase at node 21

| Bus no. | Voltage <br> Magnitude | Voltage Angle | Value of L- <br> index |
| :---: | :---: | :---: | :---: |
| 3 | 0.9350 | -14.3062 | 0.0464 |
| 4 | 0.9110 | -18.0204 | 0.0574 |
| 6 | 0.9113 | -21.4970 | 0.057 |
| 7 | 0.9228 | -22.5794 | 0.0479 |
| 9 | 0.8392 | -34.7898 | 0.2414 |
| 10 | 0.7105 | -43.9661 | 0.5472 |
| 12 | 0.9110 | -35.9393 | 0.1713 |
| 14 | 0.8672 | -38.1883 | 0.245 |
| 15 | 0.8307 | -38.7009 | 0.2949 |
| 16 | 0.8133 | -39.1530 | 0.3237 |
| 17 | 0.7329 | -42.7241 | 0.4906 |
| 18 | 0.7704 | -41.5714 | 0.4141 |
| 19 | 0.7397 | -43.1141 | 0.4824 |
| 20 | 0.7311 | -43.4040 | 0.5014 |
| 21 | 0.5639 | -50.7676 | 0.9593 |
| 22 | 0.5916 | -48.9828 | 0.8614 |
| 23 | 0.7592 | -41.1171 | 0.4274 |
| 24 | 0.6775 | -44.3805 | 0.6175 |
| 25 | 0.7646 | -38.3516 | 0.4235 |
| 26 | 0.7407 | -39.1052 | 0.4684 |
| 27 | 0.8327 | -34.7305 | 0.3064 |
| 28 | 0.9078 | -23.2520 | 0.0707 |
| 29 | 0.8076 | -36.6079 | 0.3612 |
| 30 | 0.7932 | -37.9791 | 0.398 |
| Lmax (system stability indicator) | 0.9593 |  |  |

Table 4: Voltage phasor, L-index and Lmax at voltage collapse point, due to load increase at node 26

| Bus no. | Voltage <br> Magnitude | Voltage Angle | Value of L-index |
| :---: | :---: | :---: | :---: |
| 3 | 0.9979 | -8.8390 | 0.0221 |
| 4 | 0.9841 | -10.9702 | 0.0262 |
| 6 | 0.9788 | -13.1083 | 0.0245 |
| 7 | 0.9755 | -14.8805 | 0.0289 |
| 9 | 1.0181 | -17.3225 | 0.0558 |
| 10 | 1.0008 | -19.5498 | 0.1043 |
| 12 | 1.0290 | -18.4395 | 0.0638 |
| 14 | 1.0077 | -19.5728 | 0.0933 |
| 15 | 0.9960 | -19.7247 | 0.1043 |
| 16 | 1.0095 | -19.1860 | 0.0891 |
| 17 | 0.9978 | -19.6604 | 0.1062 |
| 18 | 0.9850 | -20.4187 | 0.1233 |
| 19 | 0.9817 | -20.6237 | 0.1295 |
| 20 | 0.9857 | -20.4173 | 0.1245 |
| 21 | 0.9791 | -20.2918 | 0.1284 |
| 22 | 0.9769 | -20.3554 | 0.1309 |
| 23 | 0.9621 | -20.6826 | 0.1442 |
| 24 | 0.9262 | -21.6501 | 0.1896 |
| 25 | 0.8191 | -24.1256 | 0.3429 |
| 26 | 0.5582 | -31.6425 | 0.9683 |
| 27 | 0.8864 | -22.5236 | 0.2471 |
| 28 | 0.9649 | -14.3237 | 0.0441 |
| 29 | 0.8631 | -24.1741 | 0.2935 |
| 30 | 0.8496 | -25.3721 | 0.3245 |
| Lmax (system stability indicator) | 0.9683 |  |  |

Table 5: Voltage phasor, L-index, and Lmax at voltage collapse point, due to load increase at node 30

| Bus no. | Voltage <br> Magnitude | Voltage Angle | Value of L- <br> index |
| :---: | :---: | :---: | :---: |
| 3 | 0.9975 | -9.2693 | 0.023 |
| 4 | 0.9838 | -11.5089 | 0.0274 |
| 6 | 0.9775 | -13.8009 | 0.0268 |
| 7 | 0.9747 | -15.4936 | 0.0302 |
| 9 | 1.0223 | -17.9049 | 0.0544 |
| 10 | 1.0101 | -20.0509 | 0.1 |
| 12 | 1.0379 | -18.9183 | 0.0613 |
| 14 | 1.0182 | -20.0509 | 0.0892 |
| 15 | 1.0078 | -20.2504 | 0.0996 |
| 16 | 1.0186 | -19.6693 | 0.0856 |
| 17 | 0.9960 | -20.1521 | 0.1018 |
| 18 | 0.9923 | -20.9193 | 0.118 |
| 19 | 0.9960 | -21.1140 | 0.124 |
| 20 | 0.9916 | -20.9088 | 0.1193 |
| 21 | 0.9904 | -20.8231 | 0.1219 |
| 22 | 0.9800 | -20.9012 | 0.1239 |
| 23 | 0.9523 | -21.3243 | 0.135 |
| 24 | 0.8780 | -22.4498 | 0.1735 |
| 25 | 0.8574 | -25.5667 | 0.2877 |
| 26 | 0.8446 | -26.1336 | 0.3169 |
| 27 | 0.9568 | -27.4899 | 0.3523 |
| 28 | 0.7192 | -15.3127 | 0.055 |
| 29 | 0.6125 | -34.7600 | 0.6259 |
| 30 |  | -44.5881 | 0.9843 |
| Lmax (system stability indicator) |  |  |  |

In Fig. 5, L-index variations at nodes 21, 10 and 3 due to load increase at node 21 is given. It is clear from Fig. 5 that the Lindex of node 21 reaches value 1 as the voltage at node 21 reaches its critical point. In the same Fig. 5, Loading effect at node 21 on L-index of nearby node 10 and a far from node 3 is given. The L-Index variation at node 10 is from 0.0711 to 0.5472 and at node 3 is from 0.0167 to 0.0464 .


Fig. 5: Voltage/L-index of different buses to loading of Bus 21
Fig. 6 shows L-Index value tends to value near to 1 as voltage at node 26 is reaches to critical point due to load increase at node 26. In the same Fig. 6, loading effect at node 26 on Lindex of a nearby node 25 and a far node 3 is given. The value of L-index at node 25 varies from 0.0957 to 0.3429 and that of node 3 , varies from 0.0167 to 0.0221 .


Fig. 6: Voltage/L-index of different buses to loading of Bus 26
L-index of bus 29 , bus 30 and bus 7 varies due to load increase at node 30, as shown in Fig. 7. It is clear from the Fig. 7 that as the voltage at node 30 reaches to critical point, value of L-index at the node 30 reaches to value 1 . The Lindex value varies from 0.0234 to 0.0302 of a far bus 7 from bus 30 and for a neighboring bus 29 L-index varies from 0.1172 to 0.6259 .


Fig. 7: Voltage/L-index of different buses to loading of Bus 30

## 4. CONCLUSION

In this paper, OPP has been proposed using ILP methodology. From the proposed results it is clearly observed that index L for node in a multi-node system reaches near to 1 at the steady state of voltage collapse point. Moreover, the index L consolidates the impact of the load at the power system node it is determined, just as the stacking in alternate parts of the power system. Be that as it may, the impact of different loads relies upon how the node under thought is associated with alternate nodes. Results show that there is no huge impact on the index value for a load bus that isn't associated specifically to the bus where voltage is falling.

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